Prop: For all sets A and B we have

$$(A \setminus B) \cup (A \cap B) = A$$

pf: Let A and B be arbitrary sets.

We compute as follows:

$$
\begin{aligned}
&(A \setminus B) \cup (A \cap B) \\
&= \{x : x \in A \setminus B \text{ or } x \in A \cap B\} && (\text{Def}^{\text{n}} \text{ of union}) \\
&= \{x : (x \in A \text{ and } x \notin B) \text{ or } x \in A \cap B\} && (\text{Def}^{\text{n}} \text{ of set difference}) \\
&= \{x : (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \in B)\} && (\text{Def}^{\text{n}} \text{ of intersection}) \\
&= \{x : x \in A \text{ and } (x \notin B \text{ or } x \in B)\} && (\text{Distributivity of ``and'' over ``or''}) \\
&= \{x : x \in A \text{ and } (\text{True})\} && (\text{Law of Exclusive Middle}) \\
&= \{x : x \in A\} && (\text{Simplification: } P \wedge T \equiv P) \\
&= A
\end{aligned}
$$

Hence we conclude $(A \setminus B) \cup (A \cap B) = A$, as claimed.